SELF-SIMILAR COLLAPSE OF A SELF-GRAVITATING VISCOUS DISK

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ABSTRACT

A self-similar solution for time evolution of isothermal, self-gravitating viscous disks is found under the condition that $\alpha' \equiv \alpha(H/r)$ is constant in space (where α is the viscosity parameter and H/r is the ratio of a half-thickness to radius of the disk). This solution describes a homologous collapse of a disk via self-gravity and viscosity. The disk structure and evolution is distinct in the inner and outer parts. There is a constant mass inflow in the outer portions so that the disk has flat rotation velocity, constant accretion velocity, and surface density decreasing outward as $\Sigma \propto r^{-1}$. In the inner portions, in contrast, mass is accumulated near the center owing to the boundary condition of no radial velocity at the origin, thereby a strong central concentration being produced; surface density varies as $\Sigma \propto r^{-5/3}$. Moreover, the transition radius separating the inner and outer portions increases linearly with time. The consequence of such a high condensation is briefly discussed in the context of formation of a quasar black hole.

Subject headings: accretion, accretion disks — black hole physics — galaxies: kinematics and dynamics — gravitation — stars: formation

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1. INTRODUCTION

Quasars (QSOs) are the most powerful objects that have ever existed in the universe. The emergence of quasars at high-redshifts, $z \lesssim 5$, is thus crucial when considering the formation of astrophysical objects, notably of galaxies. The view is widely accepted that QSO phenomena result from mass accretion onto supermassive black holes. However, the formation process of seed black holes at high redshifts is not well understood at the present. There are two distinct lines of thoughts concerning this issue. One is based on considering a formation of a proto-quasar supermassive black hole after the formation of a host galaxy as the consequence of stellar mass loss and star encounters at the nucleus of the galaxy (Rees 1984). The other rather assumes a galaxy-independent population of massive black holes (Carr, Bond, & Arnet 1984; Loeb 1993; Fukugita & Turner 1996). Under the latter picture, a question is how quasar black holes formed at high redshifts, $z > 5 - 10$.

Suppose a high density fluctuation with a mass scale of $\sim 10^6 M_\odot$ began to collapse at high redshifts of $z \gtrsim 10$. Such a cloud acquires angular momentum through tidal torque in the course of a gravitational collapse. Resultantly formed a rotationally supported, self-gravitating disk. For a typical spin parameter, the angular momentum barrier is by roughly seven orders of magnitude larger than the Schwarzschild radius (Loeb 1993; Eisenstein & Loeb 1995). The problem is then how to get rid of angular momentum from the cloud so as to form a black hole. Radiation drag via the cosmic background radiation seems to have been at work at $z > 100$, but is effective only when the cloud is optically thin (Loeb 1993; Umemura, Loeb, & Turner 1993; Tsuribe & Umemura 1996). Afterwards, angular momentum in the cloud could be redistributed via gravitational torque rising from nonaxisymmetric perturbations (Paczyński 1978; Lin & Pringle 1987; Papaloizou & Lin 1989) and/or turbulent shear viscosity which could be associated with magnetic fields (Shakura & Sunyaev 1973). It is thus worth investigating how a self-gravitating, viscous disk evolves in the context of black-hole formation. Furthermore, this kind of study is of great importance, of course, when one investigates physics of galaxy and star formation.

The basic equilibrium structure of accretion disks are now well understood, as long as we believe the standard model based on the α-viscosity prescription (Shakura & Sunyaev 1973). Nevertheless, it is not easy to follow its dynamical evolution, mainly because the basic equations for the disks are highly nonlinear, especially when the disk is self-gravitating (e.g. Paczyński 1978; Fukue & Sakamoto 1992). To follow nonlinear evolution of dynamically evolving systems, in general, the technique of self-similar analyses is sometimes useful. Several classes of self-similar disk solutions were known previously (Pringle 1974; Filipov 1984),but all of them considered a disk in a fixed, external potential.

We are now concerned with dynamical evolution of a self-gravitating disk in a time-evolving, self-consistent potential. As far as steady, nonviscous rotating disks are concerned, there are plenty of works so far done. Mestel (1963) was the first to find a simple disk solution, in which physical quantities are integrated vertically with respect to the disk equatorial plane. Hayashi, Narita, & Miyama (1982) found two-dimensional, isothermal disk solutions with finite temperature (see also Toomre 1982 for stellar systems). Numerical steady solutions are calculated by several groups (Hachisu, Eriguchi, & Nomoto 1986; Bodo & Curir 1992; Hashimoto, Eriguchi, & Müller 1995). Recently, we have found a simple analytical solution for a steady, self-gravitating, isothermal disk (Mineshige & Umemura 1996, hereafter Paper I) as an extension of Mestel (1963) disk. However, little study has been done concerning dynamical evolution of self-gravitating, viscous disks.

We, in the present study, seek for a time-dependent, self-similar solution for a gravitational collapse of a rotation-supported, self-gravitating viscous disk. When a disk is sufficiently cool, gravitational instability will occur (Toomre 1964), providing a source of disk viscosity (Paczyński 1978; Lin & Pringle 1987) or causing disk fragmentation (e.g., Bodenheimer, Tohline, & Black 1980). Several authors thus mainly discussed the consequence of gravitational instability in the context of fueling to active galactic nuclei (e.g. Shore & White 1982; Shlosman & Begelman 1987; Shlosman, Frank & Begelman 1989), or (multiple) star formation (see Boss 1986; Myhill & Kaula 1992). We here adopt a rather distinct approach; we, in the present study, try to find an analytical solution for a collapse of rotating, viscous disks, putting aside for the moment the stability argument. It might be noted in this context that Shu (1977) found the self-similar solution for a gravitational collapse of an isothermal sphere. Saigo & Hanawa (1996) discussed the effects of rotation. We extend these works so as to incorporate the effects of mass accretion via viscosity. We derive self-similar solutions in section 2, and then discuss the formation of a primordial quasar black hole in section 3.

2. SELF SIMILAR, SELF-GRAVITATING DISK

2.1. Basic Equations for Self-Similar Variables

We start with the time-dependent version of the height-averaged equations for isothermal accretion disks (cf. Honma, Matsumoto & Kato 1991; Narayan & Yi 1994);

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\Sigma V_r) = 0, \tag{1}$$

$$\frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r} = -\frac{{c_\mathrm{s}}^2}{\rho}\frac{\partial \rho}{\partial r} - \frac{GM_r}{r^2} + \frac{{V_\varphi}^2}{r}, \tag{2}$$

$$\frac{\partial (rV_\varphi)}{\partial t} + V_r \frac{\partial (rV_\varphi)}{\partial r} = \frac{1}{r\Sigma}\frac{\partial}{\partial r}\left(\nu \Sigma r^3 \frac{\partial \Omega}{\partial r}\right). \tag{3}$$

Here, $\Sigma = 2\rho H$ is surface density, H is half-thickness of the disk, $\Omega = V_\varphi / R$, c_s is sound velocity (which is constant by assumption), M_r is the mass of a disk within a radius r, and we approximated a potential to be $\sim -GM_r/r$. This is a good approximation if $\Sigma(r)$ profile is steeper than $1/r$ (see Appendix). We prescribe kinematic viscosity as

$$\nu = \alpha c_\mathrm{s} H = \alpha(\frac{H}{r})c_\mathrm{s} r, \tag{4}$$

with α being viscosity parameter, because we will find later that self-similar solutions exist if $\alpha' \equiv \alpha(H/r)$ is constant in space. From now on, therefore, we assume α' (instead of α) to be constant. For vertically self-gravitating disks, H is determined as

$$H = \frac{c_\mathrm{s}}{(4\pi G\rho)^{1/2}} = \frac{{c_\mathrm{s}}^2}{2\pi G\Sigma}, \quad \rho = \frac{\Sigma}{2H} = \frac{\pi G \Sigma^2}{{c_\mathrm{s}}^2}. \tag{5}$$

To proceed, it is convenient to rewrite mass conservation (1) using $M_r(r,t)$;

$$\frac{\partial M_r}{\partial t} + V_r \frac{\partial M_r}{\partial r} = 0, \quad \frac{\partial M_r}{\partial r} = 2\pi r \Sigma. \tag{6}$$

Now, we introduce the following self-similar variables (Shu 1977);

$$x \equiv \frac{r}{c_\mathrm{s} t}, \quad \Sigma(r,t) = \frac{c_\mathrm{s}}{2\pi G t}\sigma(x), \quad M_r(r,t) = \frac{{c_\mathrm{s}}^3 t}{G} m(x),$$

$$\rho(r,t) = \frac{\sigma^2(x)}{4\pi G t^2}, \quad H(r,t) = \frac{c_\mathrm{s} t}{\sigma(x)}, \quad V_r(r,t) = c_\mathrm{s} u(x),$$

$$V_\varphi(r,t) = c_\mathrm{s} v(x), \quad j(x) \equiv xv = \frac{1}{{c_\mathrm{s}}^2}\frac{rV_\varphi(r,t)}{t}. \tag{7}$$

Note that derivatives are transformed into

$$\frac{\partial}{\partial t} \to -\frac{x}{t}\frac{\partial}{\partial x} + \frac{\partial}{\partial t'}, \quad \frac{\partial}{\partial r} \to \frac{x}{r}\frac{\partial}{\partial x}, \tag{8}$$

for the transformation, $(r,t) \to (x, t' = t)$. Since all the time derivatives with respect to t' disappear if we use self-similar variables (Eq. 7), we hereafter write d/dx instead of $\partial/\partial x$.

Equation (6) now becomes

$$m + (u - x)\frac{dm}{dx} = 0, \quad \text{and} \quad \frac{dm}{dx} = x\sigma, \tag{9}$$

yielding a simple relation between m, σ and u; $m = x\sigma(x-u)$. With this being kept in mind, equations (1) – (3) can be modified as

$$(u-x)\frac{1}{\sigma}\frac{d\sigma}{dx} + \frac{du}{dx} + \frac{u-x}{x} = 0, \tag{10}$$

$$\frac{2}{\sigma}\frac{d\sigma}{dx} + (u-x)\frac{du}{dx} - \sigma\frac{u-x}{x} - \frac{v^2}{x} = 0, \tag{11}$$

$$j + (u-x)\frac{dj}{dx} = \alpha'\frac{1}{\sigma x}\frac{d}{dx}\left[\sigma x\left(-2j + x\frac{dj}{dx}\right)\right]. \tag{12}$$

2.2. Solution in a Slow Accretion Limit

In the limit of slow accretion ($v \gg 1, \sigma \gg 1, |u| \ll 1$), equation (11) gives

$$v = \sigma^{1/2}(x-u)^{1/2}, \quad j = \sigma^{1/2}x(x-u)^{1/2}. \tag{13}$$

leading to

$$\frac{d\ln j}{d\ln x} = 1 + \frac{1}{2}\frac{1}{x-u}\left(x - \frac{du}{d\ln x}\right) + \frac{1}{2}\frac{d\ln\sigma}{d\ln x}. \tag{14}$$

Note that from equation (10) we derive

$$\frac{d\ln\sigma}{d\ln x} = \frac{1}{x-u}\frac{du}{d\ln x} - 1, \tag{15}$$

from equation (12). Inserting equation (15) into equation (14), we have

$$\frac{d\ln j}{d\ln x} = \frac{1}{2} + \frac{1}{2}\frac{x}{x-u} = \frac{2x-u}{2(x-u)}. \tag{16}$$

After some algebra, we obtain

$$\frac{u}{2x} = -\alpha'\frac{1}{\sigma x j}\frac{d}{dx}\left(\sigma x j\frac{2x-3u}{x-u}\right). \tag{17}$$

With a help of the expressions for j (Eq. 13) and σ (Eq. 15), we finally derive an ordinary differential equation for $u(x)$:

$$\frac{du}{dx} = -\frac{4x^2 - 6ux + 3u^2}{2(x-3u)x} - \frac{1}{\alpha'}\frac{u(x-u)^2}{(x-3u)x}. \tag{18}$$

Equation (18) can easily be integrated numerically for an appropriate boundary condition; $u = 0$ at $x = 0$ if we assume no central object (such as a black hole). Once $u = u(x)$ is obtained, we can derive $\sigma = \sigma(x)$ by integrating equation (15) for a given $\sigma_0 \equiv \sigma(x = 1)$. The results of the integration are displayed in figure 1 for different values of $\alpha' = 10^{-3}, 10^{-2}$, and 10^{-1}. The azimuthal velocity is derived from equation (13).

Note that each physical quantity is a rather smooth function of x. We generally find $du/dx < 0$; that is, $u(x)$ is a monotonically decreasing function of $x = r/c_{\rm s}t$. Furthermore, physical quantities, such as u and σ, are power-law functions of radius in the limits of $x \gg \alpha'$ and $x \ll \alpha'$.

In the limit of large $x(\gg \alpha')$, mass accretion is induced by viscosity. Two terms on the right-hand side of equation (18) are balanced with each other (while $du/dx = 0$). We find

$$u \approx -2\alpha', \quad \sigma \approx \sigma_0 x^{-1}, \quad v \approx \sigma_0^{1/2}, \quad \dot{m} \approx 2\alpha'\sigma_0, \tag{19}$$

where $\dot{m}(\equiv -x\sigma u)$ corresponds to a mass-flow rate. The radial dependences of physical quantities at large x are the same as those of the stationary, self-similar solution of a self-gravitating viscous disk (Paper I). However, we find $V_r \approx -2\alpha c_{\rm s}(H/r)$ in the current time-dependent solution, whereas $V_r = -\alpha c_{\rm s}(H/r)$ in the steady solution. This indicates that accretion velocity is doubled when we consider the effects of continuously growing central mass (see discussion in Paper I).

In the limit of small $x \ll \alpha'$ the first term dominates over the second on the right-hand side of equation (18),

$$u \approx -2x\left(1 - \frac{9}{11}\frac{x}{\alpha'}\right), \quad \sigma \approx \frac{\sigma_0}{\alpha'}\left(\frac{x}{\alpha'}\right)^{-5/3}\left(1 + \frac{8}{11}\frac{x}{\alpha'}\right),$$
$$v \approx (3\sigma_0)^{1/2}\left(\frac{x}{\alpha'}\right)^{-1/3}\left(1 + \frac{1}{11}\frac{x}{\alpha'}\right), \quad \dot{m} \approx 2\alpha'\sigma_0\left(\frac{x}{\alpha'}\right)^{1/3}\left(1 - \frac{1}{11}\frac{x}{\alpha'}\right). \tag{20}$$

Note that u (and therefore V_r) is not proportional to α', indicating that mass-inflow is not controlled by viscosity, but is regulated by the inner boundary condition of $V_r = 0$ at $r = 0$. Mass is thus being accumulated continuously near the origin.

To sum up, the disk structure and evolution is distinct in the inner and outer parts. The transition radius ($r_{\rm tr}$) separating these two parts increases linearly with time, because $r_{\rm tr} \approx \alpha' c_{\rm s} t \propto t$ for a fixed α' (Eq. 7). We thus assume $r_{\rm tr} = 0$ initially; in other words, we consider the later evolution of the disk with $\Sigma \propto r^{-1}$ everywhere. (This is the situation postulated in Paper I.) As matter accretes towards the center, Σ profile changes from inside.

Now we recover physical variables from self-similar ones using equation (7): we obtain

$$V_r \approx -2\alpha' c_{\rm s}, \quad \Sigma \approx \Sigma_0\left(\frac{r}{r_0}\right)^{-1}, \quad V_\varphi \simeq (2\pi G\Sigma_0 r_0)^{1/2}, \quad \dot{M} \simeq 4\pi\alpha' c_{\rm s} r_0 \Sigma_0, \tag{21}$$

at large r/t ($\gg \alpha' c_{\rm s}$), and

$$V_r \approx -2c_{\rm s}\left(\frac{r}{r_0}\right)\left(\frac{t}{t_0}\right)^{-1}, \quad \Sigma \approx \Sigma_0\left(\frac{r}{r_0}\right)^{-5/3}\left(\frac{t}{t_0}\right)^{2/3},$$

$$V_\varphi \approx c_{\rm s}\left(\frac{r}{r_0}\right)^{-1/3}\left(\frac{t}{t_0}\right)^{1/3}, \quad \dot{M} \approx 4\pi r_0 \Sigma_0 c_{\rm s}\left(\frac{r}{r_0}\right)^{1/3}\left(\frac{t}{t_0}\right)^{-1/3}, \tag{22}$$

at small r/t ($\ll \alpha' c_{\rm s}$). Here, $\dot{M} \equiv -2\pi r\Sigma V_r$ is a dimensional mass-flow rate and we approximated $M_r \approx \int^r 2\pi r_0 \Sigma_0 dr = 2\pi\Sigma_0 r_0^2$ in equation (21). The units are

$$r_0 = 1.0\ r_{\rm pc}\ {\rm pc}, \quad c_{\rm s} \simeq 10^{6.0} T_4^{1/2}\ {\rm cm\ s^{-1}}, \quad t_0 \equiv \frac{r_0}{c_{\rm s}} \sim 10^{5.0}\frac{r_{\rm pc}}{T_4^{1/2}}\ {\rm yr},$$

$$\dot{M}_0 \equiv 4\pi\alpha' c_{\rm s} r_0 \Sigma_0 \sim 10^{0.27}\frac{M_6 T_4^{1/2}}{r_{\rm pc}}\ M_\odot\ {\rm yr}^{-1}, \tag{23}$$

for temperature of $\sim 10^4 T_4$K, mass of $\sim 10^6 M_6 M_\odot$, respectively. The unit for Σ is chosen so as to give $M = \int 2\pi\Sigma(r)\ rdr$ for the initial state, in which $\Sigma = \Sigma_0 r_0/r$; For such normalizations, a normalization constant of $\sigma(x)$ is

$$\Sigma_0 = \frac{M}{2\pi r_0^2} \sim 10^{1.5}\frac{M_6}{r_{\rm pc}^2}\ {\rm g\ cm^{-2}}, \quad \sigma_0 \equiv \frac{2\pi G t_0}{c_{\rm s}}\Sigma_0 \sim 10^{1.71}\frac{M_6}{r_{\rm pc} T_4}. \tag{24}$$

Note that σ_0 represents the ratio of disk radius to height at $x = 1$ (see Eq. 7), or the initial ratio of gravitational energy to thermal energy of the disk, ${V_\varphi}^2/{c_{\rm s}}^2$ (Eq. 21). The model parameters of the self-similar solutions are α', $c_{\rm s}$ (or temperature), and σ_0.

Figure 2 plots the time evolution of a self-gravitating disk. Clearly, there are two regimes as mentioned previously (cf. Fig. 1). The radius separating the outer and inner parts is increasing linearly with time. If we follow a disk evolution at a fixed r, hence, we see that V_r is initially constant and then decreases at $t > r/\alpha' c_{\rm s}$. Accordingly, mass inflow rate also decreases with the time, causing a rapid growth of Σ and M_r. Note that since $H/r \sim (x\sigma)^{-1}$ (Eq. 7), H/r is constant at large r/t, while it rapidly decreases inward; $H/r \propto (r/t)^{2.5}$. The thin disk and slow accretion approximations are even better in the inner portions at later times, although α may exceed unity at $x \ll \alpha'$. This means, the present solution does not give a good representation of the disk structure at $r/t \ll \alpha' c_{\rm s}$ (discussed later).

3. DISCUSSION

3.1. Summary of the Self-Similar Solution

We have derived a self-similar solution for time evolution of an isothermal, self-gravitating, viscous disk in the slow accretion limit. Disk structure changes from the inner to outer parts. For example, surface density is scaled as $r^{-5/3}$ in the inner, while it is r^{-1} in the outer. This interface gradually moves outward in proportion to t. In this solution density increases monotonically with the time at the center. The mass profile near the center is

$$M_r(r) = \int_0^r 2\pi\Sigma(r)\ rdr \simeq 3\times 10^6 \left(\frac{r}{r_0}\right)^{1/3} \left(\frac{t}{t_0}\right)^{2/3} \frac{r_{\rm pc}}{T_4^{1/3}} M_\odot. \tag{25}$$

[Although this yields a diverging M_r, the increase of M_r should be terminated in a realistic situation, when the outer disk is depleted with gas.]

As claimed first by Mestel (1963) and also by Paper I, the thin-disk approximation breaks down at radii comparable with the thickness. In fact, the present solution gives diverging Ω and α as x approaching 0, which suggests that the solution does not represent physical situation at $x \ll 1$. Moreover, since we assume steady mass input towards the center, the central mass condensation increases at any time. Once a central object forms from a central mass condensation, gravity is dominated by this object at sufficiently small radii, where we may adopt a solution for a point-mass potential.

Realistically, there may be two or three zones in a disk. Before forming an object, a self-gravitating disk has two zones (as mentioned in previous section). After the formation of a central object, in contrast there are three zones; the innermost region is dominated by a point-mass potential and the other two zones are dominated by self-gravity of the disk. Since $\dot{M} > 0$, the mass of the central object is continuously increasing with time. The transition radius between the innermost to the inner region again increases linearly with time (Paper I).

Self-similar solutions assume that heating and cooling rates have the same radial dependence (see Eq. 4 in Paper I). A flat temperature distribution is the result of this assumption. This is a reasonable approximation at least in the outer regions: when we balance viscous heating and radiative cooling rates in a thin-disk approximation, we find $c_{\rm s} \propto r^{-1/12} \sim r^{-3/13}$, depending on the optical depth of the disk and opacity sources (Paper I). This relatively flat temperature profile results from the fact that for $\Sigma \propto r^{-1}$ (as in the outer parts) the potential is logarithmic and thus has a weak radial dependence. At $x \ll 1$, in contrast, this approximation may break down, since potential has stronger radial dependence. The isothermal approximation may not be justified at the innermost region at later times ($r \ll c_{\rm s} t$).

A self-gravitating disk is locally stable, if

$$Q \equiv \frac{c_{\rm s}\kappa}{\pi G \Sigma} \gtrsim 1, \qquad (26)$$

as long as the effects of viscosity and radial mass inflow are ignored (Toomre 1964).Here, κ is epicyclic frequency and $\kappa = 2^{1/2}\Omega$ for $\Omega \propto R^{-1}$. If we simply apply this criterion to the present model, we find $Q \simeq 2^{3/2}(H/R)^{1/2}$ at $x \gtrsim 1$ (Eq. 5 and 21), indicating that the disk is stable for $H/R \gtrsim 1/8$. If H/R is small, gravitational instability will set out, making disk turbulent, thickening the disk (Paczyński 1978). However, this is a very naive picture and a more sophisticated stability analysis, similar to Christodoulou et al. (1995a, 1995b) but including the effects of disk viscosity and radial gas inflow, is needed as future work.

3.2. Formation of a Quasar Black Hole

When M_r exceeds a critical value at some radius,

$$M_{\rm crit}(r) = (r/10^{5.4}{\rm cm})^{2/3} M_\odot, \qquad (28)$$

the cloud will start to collapse due to a general relativistic instability (Shapiro & Teukolsky 1983), resulting in the formation of a black hole. Equation (28) gives a critical mass (for a given radius) for spherical supermassive stars, while we are now concerned with evolution of a rotation-supported disk. Nevertheless, we employ the argument concerning spherical stars in order to see qualitative effects of general relativity, since a thin-disk approximation breaks down anyway near the center as mentioned above, and since a solid analysis of a collapsing self-gravitating disk based on the general relativistic formulation is not available at this moment.

With this being kept in mind, we discuss a fate of a rotationally supported, viscous disk with a mass of $\sim 10^6 M_\odot$, a temperature of $\sim 10^4$K, and a size of several pc. In the present picture, such a relatively high disk temperature is preferable, since otherwise the disk will stay molecular rather than ionized. The accretion timescale is inversely proportional to the temperature, and hence it may exceed the age of the Universe for a molecular disk with $\alpha < 0.01$ (e.g. Eq. 1 in Sasaki & Umemura 1996), unless alternative mechanisms, such as gravitational torque, remove the disk angular momentum. There are several possibilities to heat the disk. First, if the formation of primordial hydrogen molecules proceeds more slowly than the dynamical collapse, gas will not cool below $\sim 10^4$K. This may occur if

residual free electrons recombine quickly due to density enhancement, thereby suppressing the formation of a sufficient amount of H^- ions, which help to make hydrogen molecules (see Hutchins 1976, Palla, Salpeter, & Stahler 1983). Second, if the Universe was reionized through first-generation stars or objects, the disk will be effectively heated by strong UV background radiation (e.g. Sasaki & Umemura 1996). Finally, if star formation occurs within the disk itself, the disk material can be photoionized by stars.

Figure 3 depicts the evolution of such a disk (by the solid lines) and the critical line for a gravitation instability (by the dotted line) in the ($\log r$-$\log M_r$) diagram. As time goes on, the disk becomes more and more condensed at the center, thereby increasing its mass within a fixed radius. The mass profile is $M_r \propto r^{1/3}$ (Eq. 25) according to the self-similar solution, while the critical value gives $M_{\rm crit} \propto r^{2/3}$ (Eq. 28). The solid line should cross the dotted line at

$$r_{\rm crit} \simeq 10^{11.8} \left(\frac{t}{t_0}\right)^2 \frac{r_{\rm pc}^3}{T_4^{3/2}}\ {\rm cm}, \quad M_r(r_{\rm crit}) \simeq 10^{4.3} \left(\frac{t}{t_0}\right)^{4/3} \frac{r_{\rm pc}^2}{T_4}\ M_\odot. \tag{29}$$

We get a condensation of $\sim 10^3 M_\odot$ on a timescale of $\sim 0.1\ t_0 \sim 10^4$yr.

The estimates above are optimistic, however, since it takes $r_0/c_{\rm s} \sim 10^5$yr to $r_0/(\alpha' c_{\rm s}) = 10^6(\alpha'/0.1)^{-1}$yr for accreting gas to reach the center, and thereby establishing a self-similar evolution of the disk. We thus safely conclude that within a timescale of $\sim 10^5(\alpha')^{-1}$yr a central region with a mass of $10^{4-5} M_\odot$ could become unstable, which may give rise to a proto-quasar black hole at high redshifts. Again, a general relativistic study of a collapsing rotating disk is necessary to conclude whether this scenario can work or not.

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A. Self-gravity under a thin-disk approximation

The most straightforward expression for the potentials under thin-disk approximation is

$$\Psi(r) = 2G\int_0^\pi d\theta \int_0^{r_0} \frac{\Sigma(R)RdR}{(r^2+R^2-2rR\cos\theta)^{1/2}}, \tag{A1}$$

(e.g. Mestel 1963), where r_0 denotes the size of the disk and we ignored vertical mass distribution in the disk. After some algebra, we have

$$\frac{d\Psi}{dr} = G(I_1+I_2+I_3), \tag{A2}$$

where I_1, I_2, and I_3 represent the Keplerian term, finite contributions from the mass within R, and the mass beyond R, respectively, and are

$$\begin{aligned} I_1 &\equiv \frac{1}{r^2}\int_0^r 2\pi R\Sigma(R)dR, \\ I_2 &\equiv 2\pi\sum_{k=1}^{\infty}\alpha_{2k}\left(\frac{(2k+1)}{r^{2k+2}}\int_0^r R^{2k+1}\Sigma(R)dR - \Sigma(r)\right) \\ I_3 &\equiv 2\pi\sum_{k=1}^{\infty}\alpha_{2k}\left(\Sigma(r) - 2kr^{2k-1}\int_r^{r_0}\frac{\Sigma(R)}{R^{2k}}dR\right), \end{aligned} \tag{A3}$$

with

$$\alpha_{2k} \equiv \frac{1}{\pi}\int_0^\pi P_{2k}(\cos\theta)d\theta = \left[\frac{(2k)!}{(2^k k!)^2}\right]^2, \tag{A4}$$

(P_{2k} is the Legendre function; see Eq. 24 of Mestel 1963). When $\Sigma(r) = \Sigma_0 r_0/r$, in particular, we find

$$\frac{d\Psi}{dr} = \frac{2\pi G\Sigma_0 r_0}{r}\left[1+\sum_{k=1}^{\infty}\alpha_{2k}\left(\frac{r}{r_0}\right)^{2k}\right]. \tag{A5}$$

We, hence, understand that if $\Sigma(r)$ profile is steeper than $1/r$ we may approximate gravitational attraction force to be $-GM_r/r^2$ except near the outer edge.

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Fig. 1.— Radial profiles of the self-similar variable, $\sigma(x)$, as functions of $x \equiv r/c_{\rm s}t$. The three solid lines represent the calculated values of σ/σ_0 for $\alpha' = 10^{-3}, 10^{-2}$, and 10^{-1}, respectively, where $\sigma_0 \equiv \sigma(x = 1)$. The transition radius at $x \sim \alpha'$ separates the outer part, where $\sigma \propto x^{-1}$, and the inner part, where $\sigma \propto x^{-5/3}$, in each curve. Note that the dotted line corresponds to $\sigma/\sigma_0 = x^{-1}$.

Fig. 2.— Time evolution of a self-gravitating disk. ¿From the top to the bottom, time development of $\dot{M}$ distribution, Σ profile, and radial distributions of V_φ (by the dashed line) and V_r (by the solid line). The units are $r_0 = 1$pc, $\dot{M}_0 \sim 2\ M_\odot {\rm yr}^{-1}$, $\Sigma_0 \sim 30{\rm g\ cm}^{-2}$, $c_{\rm s} \sim 10{\rm km\ s}^{-1}$, and $t_0 \simeq 10^5$yr, respectively. Parameters are $\alpha' = 0.1$ and $\sigma_0 = 50$. The elapsed times are $t/t_0 = 0.1$ (indicated by i), 1.0, 10, and 10^2 (indicated by f), respectively.

Fig. 3.— Evolution of mass profiles of a self-gravitating disk with a total mass of $10^6 M_\odot$, a temperature of 10^4K, and a size of 1pc (by the solid lines). The attached numbers represent the elapsed times; $t/t_0 = 0.1, 1.0$, and 10. We assumed $\alpha' = 0.1$ and $\sigma_0 = 50$. Also displayed are the critical line for a general relativistic instability, $r_{\rm crit}$ (by the dotted line), and the Schwarzschild radius $r_{\rm g}$ (by the short-dashed line), respectively.